KOLMOGOROV EQUATIONS FOR STOCHASTIC REACTION-DIFFUSION EQUATIONS

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Abstract
Stochastic ordinary differential equations (SODEs) have been adopted by many authors to model the population growth problems in random environments. It is well known that the solution of a SODE is a diffusion process in finite dimension and the conditional expectation of a smooth function of the solution process satisfies a parabolic equation, known as the Kolmogorov equation. Formally such a connection also exists for a stochastic partial differential equation (SPDE), such as a stochastic reaction-diffusion equation. But the corresponding Kolmogorov equation is infinite-dimensional. In this talk, we will first show by a simple example that the Kolmogorov equation for a SPDE contains singular coefficients which rule out the existence of classical solutions. To seek weak solutions, we introduce a class of $L^2(\mu)$-Sobolev spaces with a suitable Gaussian measure $\mu$. In this Gauss-Sobolev space setting, the weak (variational) solutions to the Cauchy problems for a class of parabolic equations are formulated. It will be shown that the existence and uniqueness theorems can be proved as in finite dimension.