NON-AZIMUTHAL SOLUTIONS TO A NONLINEAR ELLIPTIC EQUATION ON A SPHERICAL CAP

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Abstract
This talk is based on the joint work with C. Bandle (University of Basel, Switzerland) and H. Ninomiya (Meiji University, Japan).
We consider the following nonlinear elliptic problem on a spherical cap
\[ \begin{align*}
\Lambda u + \lambda (u^p - u) &= 0 \quad \text{in } \Omega_\varepsilon \subset S^n, \\
u &> 0 \quad \text{in } \Omega_\varepsilon, \\
\mathcal{B}u &= 0 \quad \text{on } \partial \Omega_\varepsilon, 
\end{align*} \]
where \( \Lambda \) is the Laplace-Beltrami operator on the unit sphere \( S^n \) with \( n \geq 2 \), \( \Omega_\varepsilon \) is a spherical cap centered at the North Pole with its geodesic radius \( \pi - \varepsilon \), \( \varepsilon > 0 \) is small, \( \mathcal{B}u = \partial_n u \), or \( \mathcal{B}u = u \) with \( n \) being the outward normal differential operator to \( \partial \Omega_\varepsilon \), and \( \lambda > 0 \) is a parameter. We investigate how the bifurcations diagrams differ from that in the case \( \varepsilon = 0 \). Since we are interested in solutions which are close to 1, we consider the linearized problem around \( u \equiv 1 \):
\[ \begin{align*}
\Lambda v + (p-1)\lambda v &= 0 \quad \text{in } \Omega_\varepsilon \subset S^n, \\
\mathcal{B}v &= 0 \quad \text{on } \partial \Omega_\varepsilon. 
\end{align*} \]
We have the following theorem. Similar results hold for any \( n \geq 2 \).

Theorem. Suppose that \( n = 2 \) and \( \varepsilon > 0 \) is small. Let \( \mathcal{B}u = \partial_n u \). Then for each \( k \in \mathbb{N} \), around \( (p-1)\lambda = k(k+1) \), there exist \( (k+1) \) distinct eigenvalues \( \lambda_{k,\varepsilon,m} \) \((m = 0, 1, \ldots, k)\) to (0.2) such that
\[ (p-1)\lambda_{k,\varepsilon,m} - k(k+1) \approx c_{k,m} \varepsilon^{2m}, \quad m = 1, 2, \ldots, k, \]
\[ (p-1)\lambda_{k,\varepsilon,0} - k(k+1) \approx c_{k,0} (\log(\varepsilon/2))^{-1}, \quad m = 0, \]
with some positive constant \( c_{k,m} \).
Moreover, solutions to (0.1) bifurcate from \( \lambda = \lambda_{k,\varepsilon,m}/(p-1) \) and they depend on both the latitude and the longitude if \( m \geq 1 \) and depend only on the latitude if \( m = 0 \).
Remark. In the whole sphere $S^2$ case, the multiplicity of the eigenvalue $k(k + 1)$ is $2k + 1$. On the other hand, the multiplicity of $\lambda_{k,\varepsilon,m}$ is 2 if $m \geq 1$ and that of $\lambda_{k,\varepsilon,0}$ is 1. Thus, by the presence of $\varepsilon > 0$, the eigenvalue $k(k + 1)$ in the whole sphere case is decomposed into $(k + 1)$ eigenvalues.

If time permits, the Dirichlet case will be discussed.

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