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Title: On a Length-preserving Nonlocal Flow of Convex Closed Plane Curves

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Abstract:

We study a length-preserving, area-increasing, nonlocal flow of convex closed plane curves and show that this flow evolves such curves into round circles in $C^\infty$ norm. The evolution equation has the form

$$\frac{\partial X(u,t)}{\partial t} = \left[ k^\alpha - \frac{1}{2\pi} \int_{X(u,t)} k^{\alpha+1} ds \right] N, \quad \alpha > 1,$$

$$X(u,0) = X_0(u), \quad u \in S^1. \quad (1)$$

Here the initial data $X_0(u) \subset \mathbb{R}^2$ is assumed to be a smooth embedded convex closed curve parametrized by the parameter $u \in S^1$ with counterclockwise orientation; $k$ and $N$ are the curvature and the inward unit normal vector of the evolving curve respectively. The integral in (1) is with respect to arc length parameter.