FINITE TIME BLOWUP OF THE STOCHASTIC SHADOW GIÉRER-MEINHARDT SYSTEM

FANG LI AND LIHU XU

ABSTRACT. By choosing some special (random) initial data, we prove that with probability 1, the stochastic shadow Giérrer-Meiñhardt system blows up pointwisely in finite time. We also give a (random) upper bound for the blowup time and some estimates about this bound. By increasing the amplitude of the initial data, we can get the blowup in any short time with positive probability.

Keywords: Stochastic shadow Giérrer-Meiñhardt system, Finite time blowup, Brownian motions, Itô formula.

Mathematics Subject Classification (2000): 60H05, 60H15, 60H30.

1. INTRODUCTION

Many of the mathematical models that have been proposed for the study of population dynamics, biochemistry, morphogenesis and other fields, take the following form:

\[
\begin{cases}
\partial_t u = d_1 \Delta u + f(u, v) & \text{in } \Omega \times (0, T), \\
\tau \partial_t v = d_2 \Delta v + g(u, v) & \text{in } \Omega \times (0, T), \\
\partial_{\nu} u = \partial_{\nu} v = 0 & \text{on } \partial \Omega \times (0, T),
\end{cases}
\]

where \( \Delta = \sum_{i=1}^{n} \partial_{x_i}^2 \) is Laplace operator, \( \Omega \) is a bounded smooth domain in \( \mathbb{R}^n \) with unit outward normal vector \( \nu \) on its boundary \( \partial \Omega \); the two positive constants \( d_1, d_2 \) are the diffusion rates of two substances \( u \) and \( v \) respectively; \( \tau > 0 \) is the number tuning response rate of \( v \) related to the change of \( u \); \( f, g \) are both smooth functions referred to as the reaction terms.

As we choose

\[
f(u, v) = \frac{u^p}{v^q}, \quad g(u, v) = \frac{u^r}{v^s},
\]

with \( p > 0, q > 0, r > 0, s \geq 0 \) satisfying the condition:

\[
0 < \frac{p - 1}{r} < \frac{q}{s + 1},
\]

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