Analysis on a Diffusive SIS Epidemic Model with Logistic Source

Bo Li, a* Huicong Li, b† and Yachun Tong c†

a. School of Mathematics and Statistics, Jiangsu Normal University,
   Xuzhou, 221116, Jiangsu Province, China.

b. Center for Partial Differential Equations, East China Normal University,
   500 Dongchuan Road, Minhang, 200241, Shanghai, China.

c. School of Mathematics and Statistics, Jiangsu Normal University,
   Xuzhou, 221116, Jiangsu Province, China.

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Abstract

In this paper, we are concerned with an SIS epidemic reaction-diffusion model with logistic source in spatially heterogeneous environment. We first discuss some basic properties of the parabolic system, including the uniform upper bound of solutions and global stability of the endemic equilibrium when spatial environment is homogeneous. Our primary focus is to determine the asymptotic profile of endemic equilibria (when exist) if the diffusion (migration) rate of the susceptible or infected population is small or large. Combined with the results of [13] where the case of linear source is studied, our analysis suggests that varying total population enhances persistence of infectious disease.

Keywords. SIS epidemic reaction-diffusion model; logistic source; endemic equilibria; small/large diffusion; asymptotic profile.

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1 Introduction

We consider the following susceptible-infected-susceptible epidemic system:

\[
\begin{align*}
\frac{\partial S}{\partial t} - d_S \Delta S &= a(x)S - b(x)S^2 - \beta(x) \frac{SI}{S + I} + \gamma(x)I, \quad x \in \Omega, \ t > 0, \\
\frac{\partial I}{\partial t} - d_I \Delta I &= \beta(x) \frac{SI}{S + I} - \gamma(x)I, \quad x \in \Omega, \ t > 0, \\
\frac{\partial S}{\partial \nu} = &\frac{\partial I}{\partial \nu} = 0, \quad x \in \partial \Omega, \ t > 0, \\
S(x, 0) = S_0(x), \ I(x, 0) = I_0(x), \quad x \in \Omega.
\end{align*}
\]

Here, $S$ and $I$, respectively, stand for the density of susceptible and infected individuals; $d_S$ and $d_I$ are positive constants measuring the motility of susceptible and infected populations respectively; $\beta$