STABLE WEAK SOLUTIONS OF WEIGHTED NONLINEAR ELLIPTIC EQUATIONS

Xia Huang
Department of Mathematics and Center for Partial Differential Equations
East China Normal University, China

Abstract
This paper deals with the weighted nonlinear elliptic equation
\[
\begin{cases}
-\text{div}(|x|^\alpha \nabla u) = |x|^\gamma e^u & \text{in } \Omega, \\
u = 0 & \text{on } \partial \Omega,
\end{cases}
\]
where \(\alpha, \gamma \in \mathbb{R}\) satisfy \(N + \alpha > 2\) and \(\gamma - \alpha > -2\), and the domain \(\Omega \subset \mathbb{R}^N (N \geq 2)\) is bounded or not. Moreover, when \(\alpha \neq 0\), we prove that, for \(N + \alpha > 2, \gamma - \alpha \leq -2\), the above equation admits no weak solution. We also study Liouville type results for the equation in \(\mathbb{R}^N\).

Keywords
stability, weak solution, weighted sobolev space, liouville theorems, exponential nonlinearity.